

6

Process capability analysis

In general, process capability indices have been quite controversial. (Ryan, 2000, p. 186)

Overview

Capability indices are widely used in assessing how well processes perform in relation to customer requirements. The most widely used indices will be defined and links with the concept of sigma quality level established. Minitab facilities for capability analysis of both measurement and attribute data will be introduced.

6.1 Process capability

6.1.1 Process capability analysis with measurement data

Imagine that four processes produce bottles of the same type for a customer who specifies that weight should lie between 485 and 495 g, with a target of 490 g. Imagine, too, that all four processes are behaving in a stable and predictable manner as indicated by control charting of data from regular samples of bottles from the processes. Let us suppose that the distribution of weight is normal in all four cases, with the parameters in Table 6.1. The four distributions of weight are displayed in Figure 6.1, together with reference lines showing lower specification limit (LSL), upper specification limit (USL) and Target (T). How well are these processes performing in relation to the customer requirements?

In the long term the fall-out, in terms of nonconforming bottles, would be as shown in the penultimate column of Table 6.1. The fall-out is given as number of parts bottles) per million (ppm) that would fail to meet the customer specifications. The table in Appendix 1 indicates that these fall-outs correspond to sigma quality levels of 4.64, 3.50, 2.81 and 3.72 respectively for lines 1–4. Scrutiny of the distributions (the voices of the processes) with reference to the specification limits (the voice of the customer) reveals the following points:

Table 6.1 Parameters for the distributions of weight with fall-out and sigma quality level (SQL).

| Process | Mean | Standard deviation | Fall-out (ppm) | SQL |
|---------|------|--------------------|----------------|------|
| Line 1 | 490 | 1.5 | 858 | 4.64 |
| Line 2 | 492 | 1.5 | 22 752 | 3.50 |
| Line 3 | 490 | 3.0 | 95 581 | 2.81 |
| Line 4 | 487 | 0.9 | 13 134 | 3.72 |

- Line 1 is performing as well as it can with the process mean ‘on target’.
- Line 2 could perform as well as line 1 if the mean could be adjusted down from 492 to the target of 490. Adjustment of process location can often be a relatively easy thing to achieve.
- Line 3 is performing as well as it can with the process mean on target but it is inferior to lines 1 and 2 because of its greater variability. Reduction of variability would be required to improve the performance of line 3, and this can often be a relatively difficult thing to achieve.
- Line 4, although currently performing less well than line 1, has the potential to give the lowest fall-out of all four processes if the mean can be adjusted upward from 487 to 490.

Sigma quality levels are intended to encapsulate process performance in a single number. However, one must beware the danger of judging a process purely on the basis of its sigma

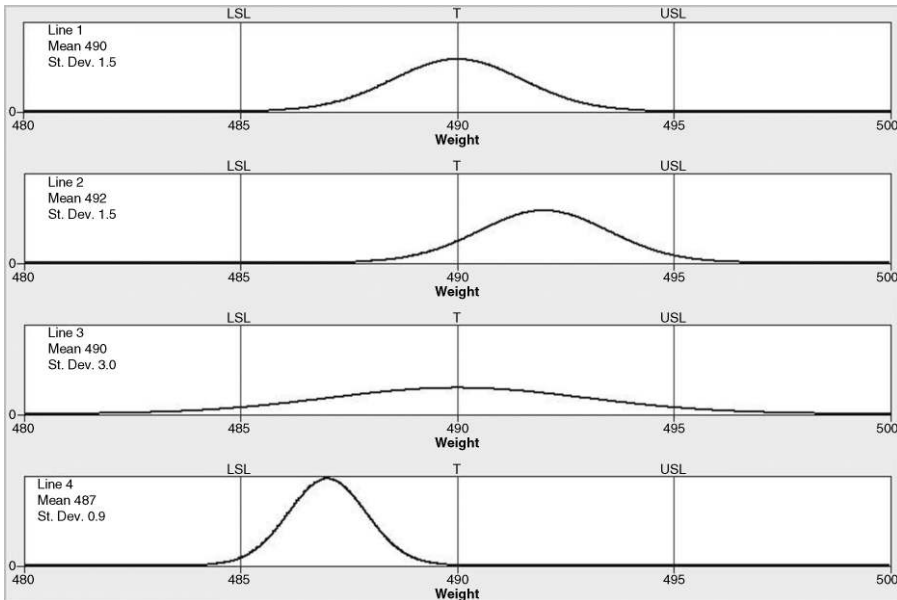


Figure 6.1 Distributions of weight for the four processes.

The process capability index C_p is given by

$$C_p = \frac{\text{Customer tolerance range}}{\text{Natural tolerance range}} = \frac{USL - LSL}{6\sigma}.$$

For line 1 this gives

$$C_p = \frac{495 - 485}{6 \times 1.5} = \frac{10}{9} = 1.11.$$

Box 6.1 Calculation of C_p for line 1.

quality level. In the case of the above four processes, reliance solely on sigma quality level could blind one to the high potential performance of line 4.

Process capability indices are designed to do the same as sigma quality levels, to encapsulate in single numbers process performance with respect to customer requirements. They can be said to measure the extent to which the ‘voice of the process’ is aligned with the ‘voice of the customer’. The fundamental fact which underpins the indices is that 99.73% of observations from a normal distribution lie between $\mu - 3\sigma$ and $\mu + 3\sigma$, i.e. in a range of three standard deviations on either side of the mean. These values are often referred to as the *natural tolerance limits* for the process. Note that the proportion 0.27% of observations will lie outside the natural tolerance range in the case of a normal distribution. The *customer tolerance range* is the range of values that the customer will tolerate, i.e. from the lower specification limit to the upper specification limit. The process capability index C_p is defined as the ratio of the customer tolerance range to the natural tolerance range. Its calculation for line 1 is displayed in Box 6.1.

The reader is invited to perform the calculations for the other three lines and to confirm the entries in Table 6.2 for the four processes. Note that line 4 ‘tops the league’ in terms having the highest C_p value, lines 1 and 2 have the same intermediate value and line 3 has the lowest. The index C_p measures the *potential* capability of a process. Thus, although lines 1 and 2 have the same potential capability, their actual capability in terms of fall-out and SQL values differs because line 2 is not operating on target. Thus a disadvantage of the index C_p is that it does not take process location into account. The process capability index C_{pk} does take process location into account. Its calculation for line 2 is displayed in Box 6.2.

The reader is invited to perform the calculations for the other three lines and to confirm the entries in Table 6.3 for the four processes.

Table 6.2 SQL and C_p values for the four lines.

| Process | Mean | Standard deviation | SQL | C_p |
|---------|------|--------------------|------|-------|
| Line 1 | 490 | 1.5 | 4.64 | 1.11 |
| Line 2 | 492 | 1.5 | 3.50 | 1.11 |
| Line 3 | 490 | 3.0 | 2.81 | 0.56 |
| Line 4 | 487 | 0.9 | 3.72 | 1.85 |

The process capability index C_{pk} is given by

$$C_{pk} = \min[C_{pl}, C_{pu}] = \min\left[\frac{\mu - LSL}{3\sigma}, \frac{USL - \mu}{3\sigma}\right].$$

For line 2 this gives

$$C_{pk} = \min\left[\frac{492 - 485}{3 \times 1.5}, \frac{495 - 492}{3 \times 1.5}\right] = \min\left[\frac{7}{4.5}, \frac{3}{4.5}\right] = \min[1.56, 0.67] = 0.67.$$

Box 6.2 Calculation of C_{pk} for line 2.

The index C_{pk} measures the *actual* capability of a process. In the type of scenario discussed here, $C_p = C_{pk}$ when the process mean coincides with the target value mid-way between the LSL and USL, i.e. when the process is centred. (Another benefit of the C_{pk} index is that it may be calculated in situations where there is only one specification limit, e.g. a customer requirement could be that a tensile strength has to be at least 25 N/mm² or that cycle time must be no greater than 40 minutes.)

Table 6.4 gives values of C_p and C_{pk} for C_p ranging from 0.5 to 2.0, first with the process centred and second with the process off centre by a 1.5 standard deviation shift. Also given are the corresponding fall-out counts of nonconforming product in ppm and the sigma quality levels. In particular, note that a Six Sigma process corresponds to a C_p value of 2.0 and a C_{pk} value no less than 1.5.

In the above discussion of the four lines it was assumed that we had perfect knowledge of the process behaviour. We now turn to the assessment of process capability in a situation where the capability indices have to be estimated from process data.

In Exercise 3 in Chapter 2, reference was made to bottle weight data stored in the worksheet Bottles.MTW. The data were collected as subgroups of size 4 and Xbar and R charts are shown in Figure 6.2. The Minitab default and recommended pooled standard deviation method for estimating the process standard deviation was used, yielding 2.039 15.

All available tests for evidence of special cause variation were applied. No signals were obtained from the charts so it appears that the process was behaving in a stable, predictable manner. Thus it is reasonable to stack the data into a single column and consider it as a sample of 100 observations from the distribution of bottle weight.

The normal probability plot in Figure 6.3 indicates that a normal distribution provides an adequate model for the data. Figure 6.4 shows a histogram of the 100 observations of weight

Table 6.3 C_p and C_{pk} values for the four lines.

| Process | Mean | Standard deviation | C_p | C_{pk} |
|---------|------|--------------------|-------|----------|
| Line 1 | 490 | 1.5 | 1.11 | 1.11 |
| Line 2 | 492 | 1.5 | 1.11 | 0.67 |
| Line 3 | 490 | 3.0 | 0.56 | 0.56 |
| Line 4 | 487 | 0.9 | 1.85 | 0.74 |

Table 6.4 C_p and C_{pk} , with fall-out rates and sigma quality levels.

| Process centred | | | Process off-centre by 1.5σ | | | Sigma quality level |
|-----------------|----------|----------------|-----------------------------------|----------|----------------|---------------------|
| C_p | C_{pk} | Fall-out (ppm) | C_p | C_{pk} | Fall-out (ppm) | |
| 0.50 | 0.50 | 133 614 | 0.50 | 0.00 | 501 350 | 1.5 |
| 0.60 | 0.60 | 71 861 | 0.60 | 0.10 | 382 572 | 1.8 |
| 0.70 | 0.70 | 35 729 | 0.70 | 0.20 | 274 412 | 2.1 |
| 0.80 | 0.80 | 16 395 | 0.80 | 0.30 | 184 108 | 2.4 |
| 0.90 | 0.90 | 6 934 | 0.90 | 0.40 | 115 083 | 2.7 |
| 1.00 | 1.00 | 2 700 | 1.00 | 0.50 | 66 811 | 3.0 |
| 1.10 | 1.10 | 967 | 1.10 | 0.60 | 35 931 | 3.3 |
| 1.20 | 1.20 | 318 | 1.20 | 0.70 | 17 865 | 3.6 |
| 1.30 | 1.30 | 96 | 1.30 | 0.80 | 8 198 | 3.9 |
| 1.40 | 1.40 | 27 | 1.40 | 0.90 | 3 467 | 4.2 |
| 1.50 | 1.50 | 6.8 | 1.50 | 1.00 | 1 350 | 4.5 |
| 1.60 | 1.60 | 1.6 | 1.60 | 1.10 | 483 | 4.8 |
| 1.70 | 1.70 | 0.34 | 1.70 | 1.20 | 159 | 5.1 |
| 1.80 | 1.80 | 0.067 | 1.80 | 1.30 | 48 | 5.4 |
| 1.90 | 1.90 | 0.012 | 1.90 | 1.40 | 13 | 5.7 |
| 2.00 | 2.00 | 0.002 | 2.00 | 1.50 | 3.4 | 6.0 |

with a fitted normal curve superimposed and reference lines indicating the lower specification limit of 485 and upper specification limit of 495. The target value for weight will be considered to be $T=490$, the mid-point of the specification range. The diagram gives a visual representation of process capability and indicates that some of the bottles measured failed to meet

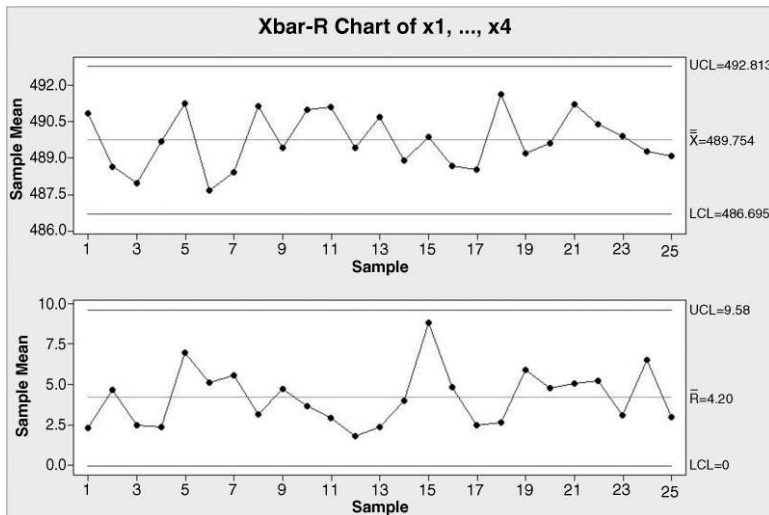


Figure 6.2 Xbar and R charts of bottle weight data.

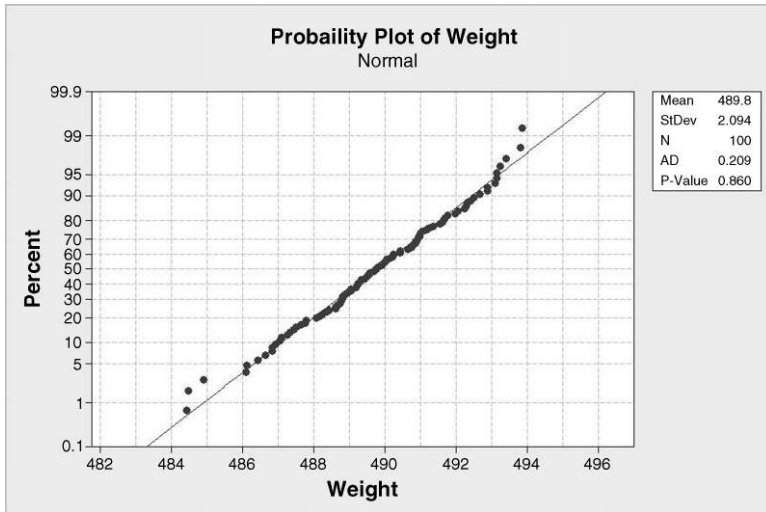


Figure 6.3 Normal probability plot of the stacked data.

customer requirements through being too light. One way in which we can proceed to estimate process capability is as detailed in Box 6.3.

Many authors deem a process with a C_p value less than 1 to be incapable. The fact that C_{pk} is less than C_p indicates that the process is not centred. The estimates of process mean and standard deviation may be used to predict fall-out for the process, as it is currently operating, at 14 913 ppm, with 9866 ppm predicted to be below the LSL and 5047 ppm predicted to be above the USL.

In order to perform the capability analysis using Minitab one can use **Stat > Quality Tools > Capability Analysis > Normal...** Under **Options...** the **Target** was specified as

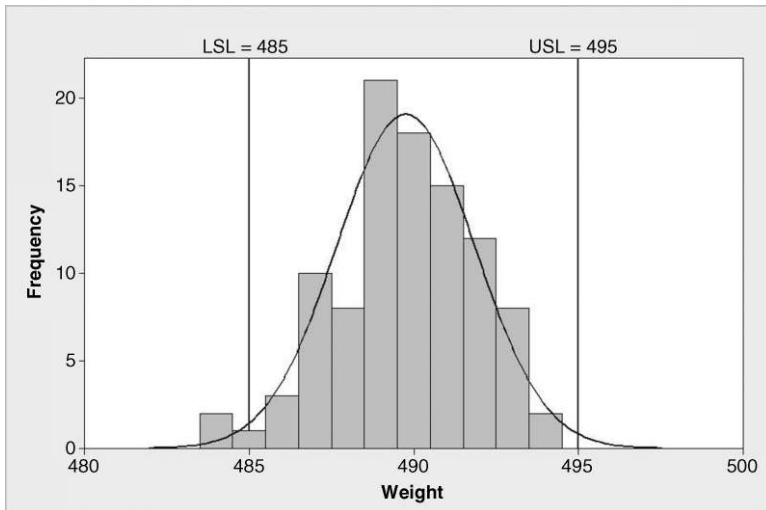


Figure 6.4 Histogram of weight with fitted normal curve and specification limits.

From the Xbar chart, $\bar{\bar{X}} = 489.754$, so we can take an estimate of the process mean to be $\mu = 489.754$. The pooled standard deviation estimate of the process standard deviation is $\hat{\sigma} = 2.039$. We then obtain

$$C_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{495 - 485}{6 \times 2.039} = \frac{10}{12.234} = 0.82,$$

$$C_{pk} = \min[C_{pl}, C_{pu}] = \min\left[\frac{\mu - LSL}{3\hat{\sigma}}, \frac{USL - \mu}{3\hat{\sigma}}\right]$$

$$= \min\left[\frac{489.754 - 485}{3 \times 2.039}, \frac{495 - 489.754}{3 \times 2.039}\right]$$

$$= \min\left[\frac{4.754}{6.117}, \frac{5.246}{6.117}\right] = \min[0.78, 0.86] = 0.78.$$

Box 6.3 Estimation of C_p and C_{pk} for bottle weight.

490. Under **Estimate...** the default **Pooled standard deviation** was chosen as the means of estimating process standard deviation in order to be able to compare Minitab output directly with the results calculated in Box 6.3 and the option **Use unbiasing constants to calculate overall standard deviation** was checked. Defaults were accepted otherwise. The dialog box is shown in Figure 6.5, with the arrangement of the data indicated and specification limits

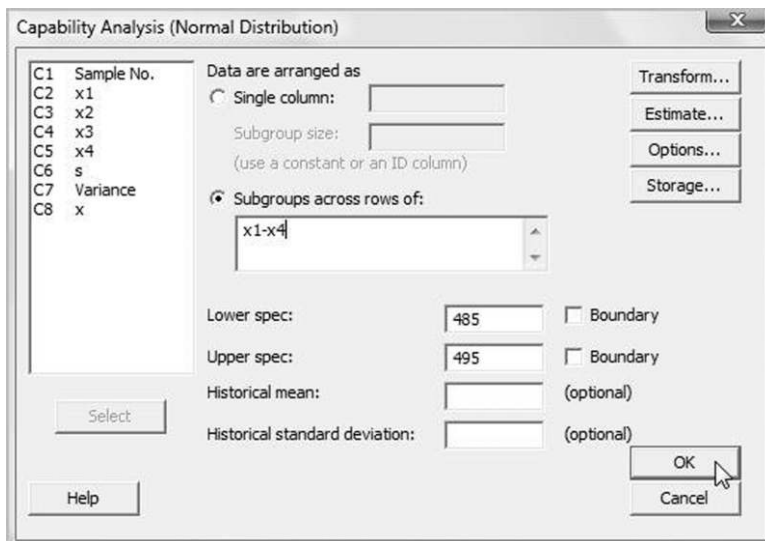


Figure 6.5 Dialog for process capability analysis via Minitab.

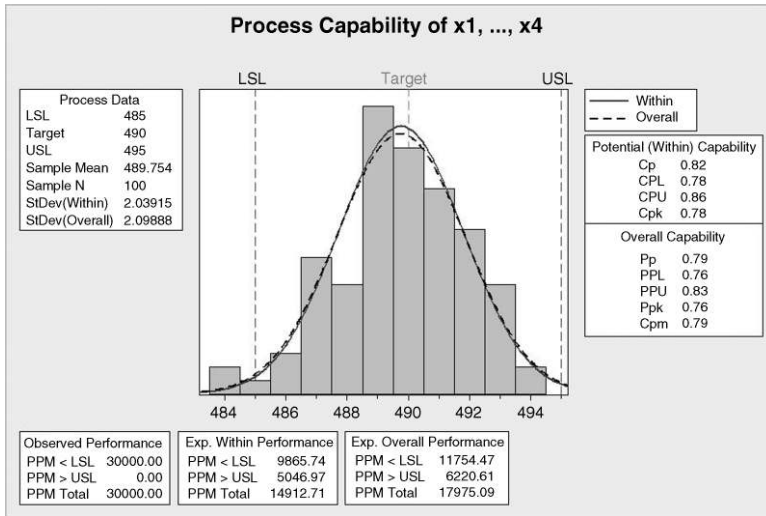


Figure 6.6 Process capability output.

entered. The output is displayed in Figure 6.6. (Process Capability™ is a trademarked feature of Minitab.)

The output in Figure 6.6 will now be considered in detail.

- The histogram of the 100 (25 subgroups of 4 bottles) bottle weight values has superimposed on it reference lines indicating the target of 490 and the LSL and USL of 485 and 495, respectively. Also superimposed are two fitted normal distributions labelled Within (solid curve) and Overall (dashed curve). Further reference will be made to these distributions below.
- In the top left-hand corner of the output the text box labelled Process Data includes two standard deviations – within and overall. These are two estimates of the process standard deviation σ . The value 2.039 15 was obtained using the pooled standard deviation method. Since this method of estimating the process standard deviation is based on the 25 subgroup standard deviations, and since standard deviation measures variability *within* subgroups, it is natural to refer to the estimate in this way. If the subgroups are stacked into a single column then **Descriptive Statistics** gives the standard deviation of the overall data set as $s = 2.093\ 59$. However, although sample variance s^2 provides an unbiased estimate of σ^2 , sample standard deviation s provides a biased estimate of σ . An unbiased estimate is obtained by dividing s by c_4 , a constant whose value depends on sample size. For sample size 100 the value of c_4 is 0.997 48 and division of 2.093 59 by this value yields 2.098 88, the value referred to as the *overall* estimate of the process standard deviation.
- The within normal curve corresponds to the $N(489.754, 2.039\ 15^2)$ distribution and the overall to the $N(489.754, 2.098\ 88^2)$ distribution.
- Three bottles from the 100 bottles measured had weight less than the LSL and none had weight above the USL. This is equivalent to a total of 30 000 ppm failing to meet the

specifications as recorded in the text box labelled Observed Performance at the bottom left of the display.

- The text boxes to the right of the one recording Observed Performance give the expected performances calculated using the within and overall normal distributions. Since the overall standard deviation exceeds the within standard deviation the predicted fall-out is greater for the former.
- The C_p and C_{pk} indices have been calculated using the within estimate of the process standard deviation and agree with the calculations given above in Box 6.3.
- The P_p and P_{pk} indices are referred to as *process performance indices*. They are analogous to the C_p and C_{pk} indices but are calculated using the overall standard deviation. The reader is invited to confirm the given values of P_p and P_{pk} as an exercise.

In this example the values of C_p and P_p (0.82 versus 0.79) are similar and the values of C_{pk} and P_{pk} (0.78 versus 0.76) are similar. This is typical of scenarios where the quality characteristic of interest is normally distributed and the process is behaving in a stable and predictable manner as evidenced by monitoring using control charts.

In discussing process performance indices, Montgomery refers to the recommendation that the capability indices C_p and C_{pk} should be used when a process is in a state of statistical control and that the process performance indices P_p and P_{pk} should be used when a process is not in a state of statistical control. He comments 'if the process is not in control the indices P_p and P_{pk} have no meaningful interpretation relative to process capability because they cannot predict process performance' (Montgomery, 2009, p. 363).

Step-by-step assessment of process capability in the above example involved: -

- Xbar and R control charts (Figure 6.2);
- a normal probability plot (Figure 6.3);
- a histogram with superimposed normal distribution and reference lines indicating the specification limits (Figure 6.4);
- the capability indices (Figure 6.6).

Use of **Stat > Quality Tools > Capability Sixpack > Normal...** essentially provides all of the output from these steps plus a run chart of the data for the last 25 subgroups and what is referred to as a capability plot. (Sixpack™ is a trademarked feature of Minitab.) Under **Estimate...**, the default **Pooled standard deviation** was chosen as the means of estimating process standard deviation and the option **Use unbiasing constants to calculate overall standard deviation** was checked. Defaults were accepted otherwise. **Perform all eight tests** was selected under **Tests...**, and **Options...** was used to specify the **Target** as 490. The source of the data and the specification limits were indicated as before. The output for the data set considered above is shown in Figure 6.7. Note that in this case there were only 25 subgroups so all of the 100 data values are displayed in the run chart.

The capability plot consists of three line segments. The lower (labelled Specs) indicates the customer tolerance range from the lower specification limit to the upper specification limit and has a tick at its midpoint, representing the target. The middle segment (labelled Overall) represents the natural tolerance range obtained using the overall estimate of standard deviation

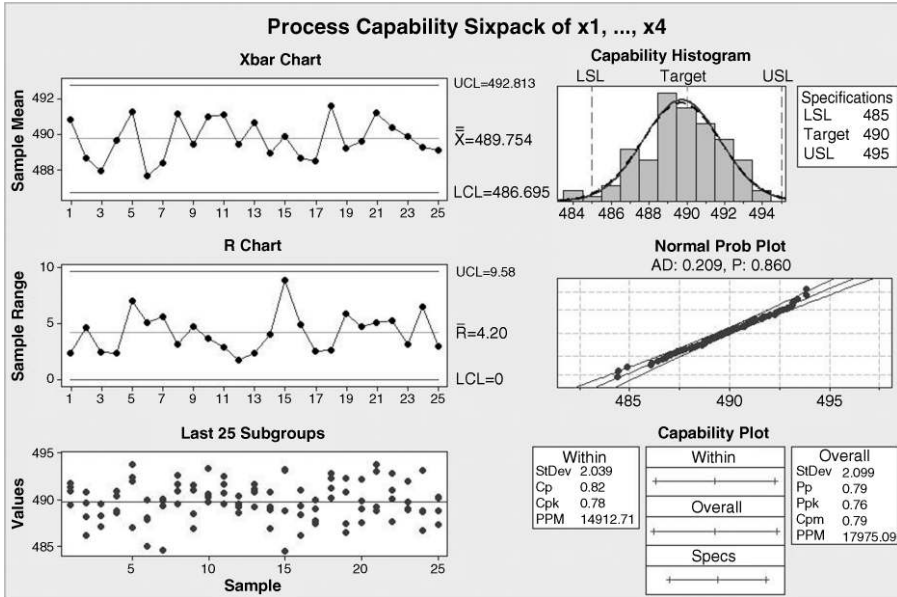


Figure 6.7 Process Capability Sixpack output.

while the upper (labelled Within) represents the natural tolerance range obtained using the within estimate of standard deviation. The index C_p is the ratio of the length of the lower segment (labelled Specs) to that of the upper segment (labelled Within); the plot gives an immediate visual indication that C_p is less than 1.

The index C_{pm} given in the output is defined as:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}$$

T is the target value, normally the mid-point of the specification range. With two-sided specification limits, the value of the C_{pk} index does not give any indication of the location of the process mean μ in relation to the specification limits LSL and USL. The C_{pm} index was developed in order to deal with this inadequacy of the C_{pk} index. If the process is centred ‘on target’ with the process mean, μ , equal to the target, T , then the C_{pm} index is identical to C_p . Minitab gives this index computed using the overall estimate of standard deviation. The main point to note is that the closer the value of C_{pm} is to the value of C_p , the closer is the process to being centred on target.

6.1.2 Process capability indices and sigma quality levels

Consider now an alternative version of the output from **Capability Analysis > Normal...** to that displayed in Figure 6.6. In order to obtain this alternative one proceeds with the dialog displayed in Figure 6.5, except that under **Options...** one selects **Benchmark Z’s (sigma level)** and checks **Include confidence intervals**. The output is shown in Figure 6.8.

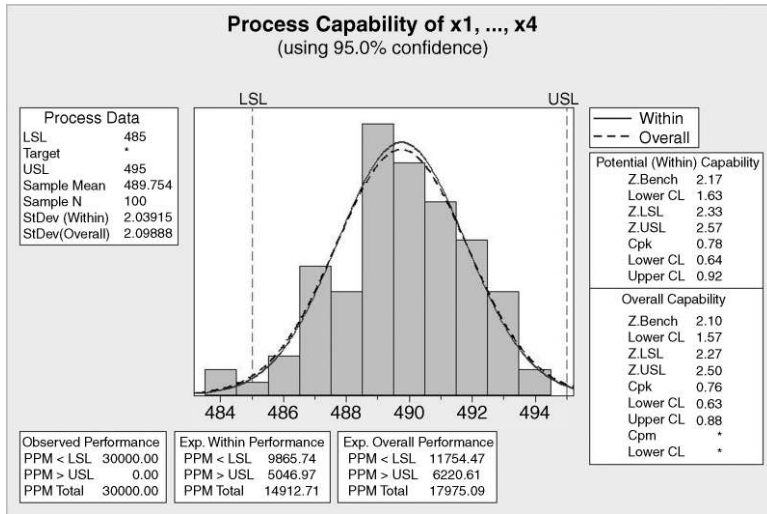


Figure 6.8 Alternative process capability output.

The expected process performance, calculated using the within estimate of the process standard deviation, is a fall-out of 14913 nonconforming bottles per million as far as specifications for weight are concerned. Reference to the table in Appendix 1 indicates that this corresponds to a sigma quality level of around 3.67. The formula in Box 6.4 may be used to convert the Z.Bench value quoted under Potential (Within) Capability to an estimate of the sigma quality level of the process. Thus the sigma quality level of the process is estimated to be $2.17 + 1.5 = 3.67$. Readers interested in the technical details of how Z.Bench is computed in Minitab should consult the Help facility.

When dealing with random variable X having mean μ and standard deviation σ , the corresponding random variable Z given by $Z = (X - \mu)/\sigma$ is referred to as the standardized random variable. Using the overall mean weight, 489.754, of the 100 bottles measured and the within estimate of standard deviation, the standardized values corresponding to specification limits, Z_1 and Z_2 are calculated in Box 6.5.

Sigma quality level = Z.Bench + 1.5

Box 6.4 Formula for sigma quality level.

$$Z_1 = \frac{USL - \mu}{\sigma} = \frac{495 - 489.754}{2.03915} = 2.57$$

$$Z_2 = \frac{LSL - \mu}{\sigma} = \frac{485 - 489.754}{2.03915} = -2.33$$

Box 6.5 Calculation of Z_1 and Z_2 .

The first of these values indicates that the USL is estimated to be 2.57 standard deviation units above the process mean, and the second indicates that the LSL is estimated to be 2.33 standard deviation units below the process mean (because of the negative sign). Note that, in the Minitab output in Figure 6.8, under Potential (Within) Capability the numerical values of these Z-values are quoted as $Z.LSL = 2.33$ and $Z.USL = 2.57$. They are used in the computation of Z.Bench within Minitab. The lower of these two values, i.e. 2.33, gives the distance to the nearest specification limit (DNS) in standard deviation units. The DNS must be at least 3 for the process to have a C_{pk} of at least 1.

It is important to bear in mind that any quoted capability index such as C_{pk} is in fact an estimate of the 'true' C_{pk} for the process. Thus, using the within estimate of process standard deviation, the estimate 0.78 was obtained for the process C_{pk} . Upper and lower 95% confidence limits for the true C_{pk} of the process are 0.64 (LCL) and 0.92 (UCL). Thus in reporting the capability analysis for bottle weight it is advisable to make the statement: 'The estimated process capability index C_{pk} is 0.78 with 95% confidence interval (0.64, 0.92).' Confidence intervals of this sort are such that they capture the true value of that which is being estimated from the data 95 times out of 100 in the long term. The value 0.78 may be thought of as a *point estimate* of C_{pk} for the process and (0.64, 0.92) may be thought of as an *interval estimate* of C_{pk} for the process. Confidence intervals will be considered in more detail in Chapter 7.

Note that in the case of Z.Bench only a Lower CL value of 1.63 is quoted. A Z.Bench value of 1.64 corresponds to a sigma quality level of $1.63 + 1.5 = 3.13$. Thus in reporting the sigma quality level for bottle weight it is advisable to make the statement: 'The estimated sigma quality level is 3.67 and it can be stated with 95% confidence that the sigma quality level is at least 3.13.'

There are situations where an assessment of process capability is required from data obtained from a single sample of product. In such situations the customer would be wise to seek assurance from the supplier that the sample to be used was taken while the process was operating in a stable and predictable manner and that the sample is representative of the population of product. Montgomery (2009, p. 348) gives such data for the burst strength (psi) of a sample of 100 bottles. The data are provided in the worksheet Burst.MTW and are reproduced by permission of John Wiley & Sons, Inc., New York. This data set will be used to illustrate a situation where there is only one specification limit, in this case a lower specification limit of 200 psi. The output from use of **Stat > Quality Tools > Capability Analysis > Normal...** with subgroup size specified as 100 is shown in Figure 6.9. Clicking on **Estimate...**, the default **Pooled standard deviation** method was checked under **Methods of estimating within subgroup standard deviation** and the option **Use unbiasing constants to calculate overall standard deviation** also selected. Under **Options...**, both **Benchmark Z's (sigma level)** and **Include confidence intervals** were selected.

The C_{pk} value is 0.67 with 95% confidence interval (0.55, 0.78). (It should be noted that in this case the within and overall estimates of the process standard deviation are identical as there is only a single sample.) The predicted fall-out based on a normal distribution of burst strength is 22 983 ppm. Appendix 1 indicates that this corresponds to a sigma quality level of around 3.5 for the process. Addition of 1.5 to the Z.Bench value of 2.0 confirms the sigma quality level of 3.5.

6.1.3 Process capability analysis with nonnormal data

Consider now data stored in columns C1 to C5 of the worksheet Density.MTW giving density measurements (g/m^2) for 80 consecutive hourly samples of size $n = 5$ from a process for the

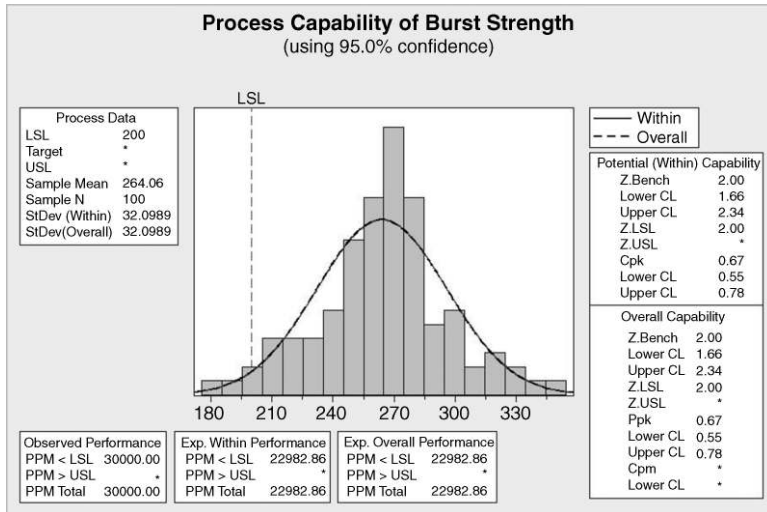


Figure 6.9 Capability analysis of burst strength.

fabrication of plastic sheeting. The specification limits are 45 to 55 g/m². Naïve use of **Stat > Quality Tools > Capability Analysis > Normal...** yields a C_{pk} of 1.29, which borders on the widely recommended minimum of 1.33. The predicted fall-out is of the order of 90 ppm. However, use of **Stat > Quality Tools > Capability Analysis > Capability Sixpack > Normal...** reveals a normal probability plot with strong evidence that the distribution of density is nonnormal (P -value less than 0.005). In situations such as this one can employ **Capability Sixpack (Nonnormal Distribution)** to investigate alternative probability distributions to the normal, such as the Weibull. In this case the Weibull probability distribution provides a satisfactory model, with a P -value of 0.195. Subsequent use of **Capability Analysis (Nonnormal Distribution)**, with selection of Weibull, gives a predicted fall-out of the order of 3000 ppm based on the fitted Weibull distribution (see Figure 6.10).

Reference to Table 6.4 indicates that this level of fall-out for a scenario where the distribution was normal, and the process was stable, predictable and centred, would correspond to C_p and C_{pk} values less than 1. Thus, in general parlance, the data indicate that the process is not capable.

Another method for dealing with data that do not have a normal distribution is to seek a transformation that will yield a new variable that is at least approximately normally distributed. Minitab provides a facility for implementing Box–Cox transformations in which the original random variable Y is transformed to $W = Y^\lambda$ when $\lambda \neq 0$ and to $W = \ln(Y)$, the natural logarithm of Y , when λ is zero. The user may either specify a value for λ or implement a procedure within the software to select an optimum value for λ .

Consider the data in the worksheet *Roughness.MTW* which gives roughness measurements (nm), for a sample of 200 machined automotive components. The upper specification limit is 800 nm. Naïve use of **Capability Analysis (Normal Distribution)**, with the data considered as a single subgroup of 200, yields a C_{pk} of 2.03 and a sigma quality level of 7.59. However, scrutiny of the histogram in the output suggests that the distribution of roughness is nonnormal. In order to carry out a capability analysis of the data following a Box–Cox

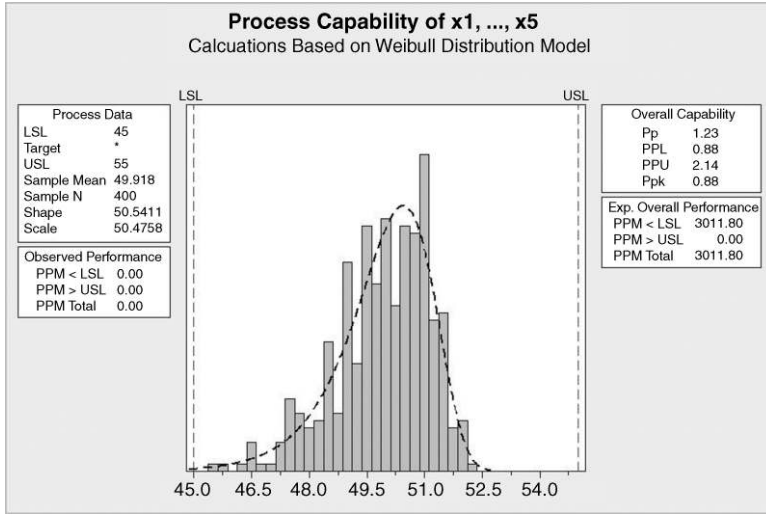


Figure 6.10 Capability analysis of density using a Weibull distribution.

transformation, use **Capability Analysis (Normal Distribution)**, select **Transform...** and check both **Box-Cox power transformation** and **Use optimal lambda** as indicated in Figure 6.11.

We have already seen that, in the case of a single subgroup, the within and overall analyses are identical (Figure 6.9). Thus under **Options...** one can uncheck **Overall analysis**. **Benchmark Z's (sigma level)** and **Include confidence intervals** were checked under **Options...** in order to obtain the output in Figure 6.12.

The heading in the output indicates that the Box-Cox transformation selected employed $\lambda = -1$. This means that the roughness values Y were replaced by $W = Y^{-1}$, i.e. by their reciprocals. Thus, for example, the roughness values 200, 500 and 800, the USL, would be

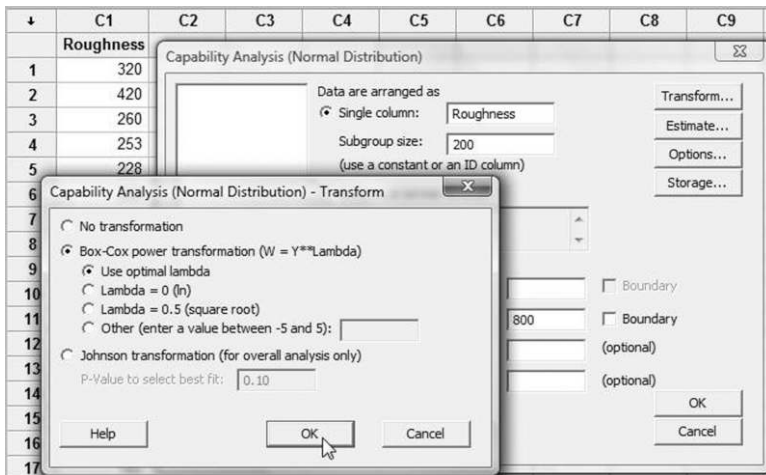


Figure 6.11 Capability analysis using a Box-Cox transformation.

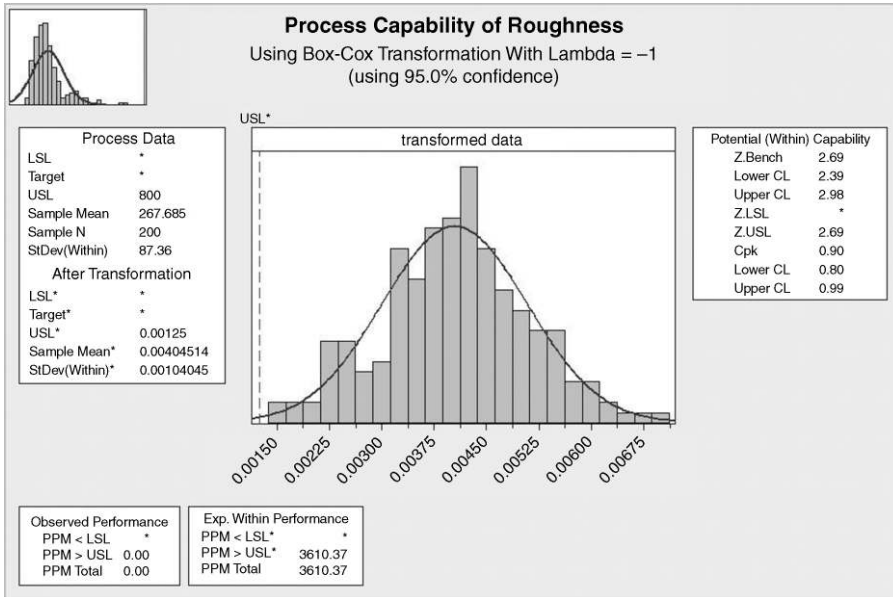


Figure 6.12 Capability analysis of the transformed roughness data.

replaced by 0.005, 0.002 and 0.00125 respectively. The USL of 800 for Y corresponds to an LSL of 0.00125, denoted by USL^* in the output, for the transformed random variable W . The histogram of the transformed data appears to indicate a normal distribution and should be compared with the histogram of the raw data shown in the top left-hand corner of the output. (The reader is encouraged to use the **Calc** menu to compute a column of the reciprocals, W , of the roughness values, Y , to perform a normality test in order to confirm that the transformation has indeed been effective and to perform the capability analysis directly on the transformed data. It has to be borne in mind that roughness less than 800 is equivalent to the reciprocal of roughness exceeding 0.00125.) However, the key information from the output is that the C_{pk} is 0.90 with 95% confidence interval (0.80, 0.99). Also the sigma quality level predicted by the analysis is $2.69 + 1.5 = 4.19$ with 95% confidence interval given by $(2.39 + 1.5, 2.98 + 1.5) = (3.89, 4.48)$.

6.1.4 Tolerance intervals

Consider a process for the production of an electronic component with a target capacitance of 2000 nF and specification limits of 1900 and 2100 nF. Suppose that the process currently yields components with capacitances that are normally distributed with mean $\mu = 2025$ nF and standard deviation $\sigma = 50$ nF. For a normal distribution 99% of values lie between $\mu - 2.58\sigma$ and $\mu + 2.58\sigma$, which in this case would be 1896 and 2154 nF. The interval (1896, 2154) is the 99% tolerance interval for capacitance, and we may refer to the interval (1900, 2100) as the specification interval. These intervals are displayed in Figure 6.13. The fact that the tolerance interval is wider than the specification interval gives an immediate indication of poor process capability. The reader may readily verify that C_{pk} is 0.50 and C_p is 0.67.

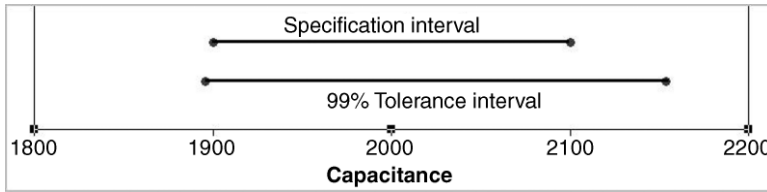


Figure 6.13 Specification interval and 99% tolerance interval.

Two steps could be taken to improve the process capability:

- Shift the mean from 2180 closer to the target value of 2000.
- Reduce the variability – a reduction in variability will result in a lower standard deviation.

Shifting a process mean is often a relatively easy task requiring, for example, a simple process adjustment. On the other hand, reduction of variability is generally a much more difficult task and may require major modifications to a process.

Suppose that, following major process changes, a sample of 150 capacitors was taken with the process operating in a state of statistical control. The measured capacitance values are provided in Capacitance.MTW. The mean and standard deviation are $\bar{x} = 1999.4$ and $s = 13.0$, respectively. The natural thing to do would be to estimate the 99% tolerance interval for the modified process as $\bar{x} - 2.58s$ and $\bar{x} + 2.58s$, i.e. (1966, 2033). However, since we are now using estimates of the population mean and standard deviation, a factor greater than 2.58 should be used. In order to estimate the 99% tolerance interval with 95% confidence the factor 2.86 should be used, with sample size 150, which yields the interval (1962, 2037). The intervals for the modified process are displayed in Figure 6.14. Clearly there has been a dramatic improvement – the reader is invited to verify that the estimated C_{pk} and C_p are 2.54 and 2.55, respectively.

Use of 95% confidence means that in the long term, when 99% tolerance intervals are calculated for samples from a normal distribution, 95 out of 100 calculated tolerance intervals will cover at least 99% of the population. The required factors for these calculations may be obtained from tables such as those in Hogg and Ledolter (1992, p. 453). Alternatively Minitab may be used.

Use of **Stat > Quality Tools > Tolerance Intervals...** is required. With **Samples in columns: Capacitance**, clicking on **Options...** and selecting **Confidence level: 95, Minimum percentage of population in interval: 99, Tolerance interval: Two-sided** and defaults otherwise, the output in Figure 6.15 is obtained. The display includes a histogram of the data, a normal probability plot that indicates that a normal distribution is a reasonable model,

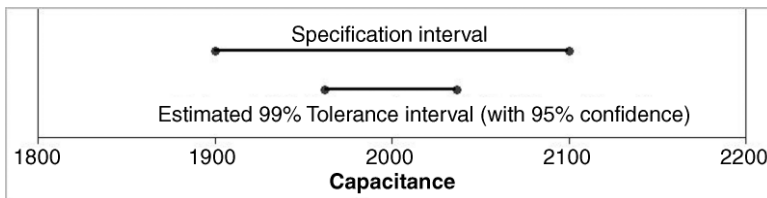


Figure 6.14 Specification interval and estimated 99% tolerance interval for modified process.

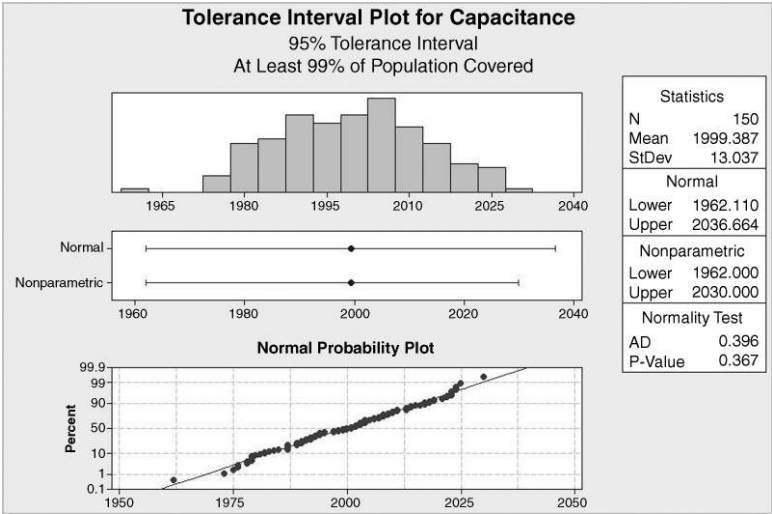


Figure 6.15 Estimated 99% tolerance interval from Minitab.

summary statistics and a 99% normal tolerance interval, after rounding, of (1962, 2037) and a 99% nonparametric tolerance interval, after rounding, of (1962, 2030). In cases where a normal distribution is an appropriate model these two intervals will be similar.

It is instructive to apply **Stat > Quality Tools > Tolerance Intervals...** to the roughness data considered at the end of the previous section. With **Samples in columns:** Roughness, clicking on **Options...** and selecting **Confidence level:** 95, **Minimum percentage of population in interval:** 99, **Tolerance interval:** Upper bound and defaults otherwise, the output in Figure 6.16 is obtained.

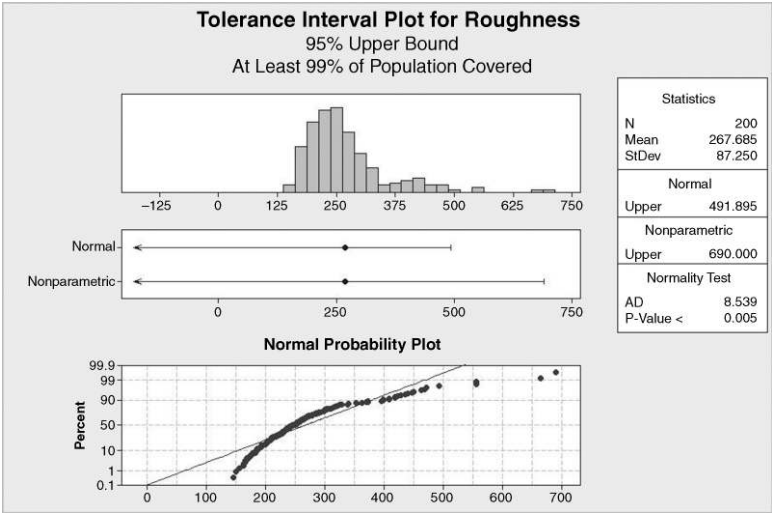


Figure 6.16 Tolerance intervals for roughness.

As was noted earlier, the normal distribution does not provide a satisfactory model for roughness. This is confirmed by the normal probability plot and associated *P*-value. The calculations based on a normal distribution estimate that at least 99% of roughness values will be less than 492, with 95% confidence. With an upper specification limit of 800 this would, on the face of it, imply a capable process. However, the nonparametric calculations, which are not based on any particular distribution, estimate that at least 99% of roughness values will be less than 690, with 95% confidence. As 690 is much closer to 800 than is 492, the conclusion is that the capability is not as good as erroneous use of the normal distribution approach would suggest.

6.1.5 Process capability analysis with attribute data

Minitab also provides capability analysis for attribute data – both for situations in which a binomial model is appropriate and for situations in which a Poisson model is appropriate. As an example of a scenario involving the binomial model, consider the data in the file Invoices1. MTW displayed in Figure 2.15 in Chapter 2, with the data for 10/01/2000 deleted, since a new inexperienced employee had processed many of the invoices during that day. Use of **Stat > Quality Tools > Capability Analysis > Binomial...** yields the output shown in Figure 6.17. In the dialog, **Defective:** was specified as No. Incomplete and **Use sizes in:** was specified as No. Invoices. Under **Tests...** the option to **Perform all four tests** was checked.

There are no signals of any special cause behaviour on the control chart of the data in the top left-hand corner of the output. Additional evidence that the process is behaving in a stable, predictable manner is provided by the display of the cumulative proportion of nonconforming invoices shown in the bottom left-hand corner of the output. It shows the cumulative proportion of nonconforming invoices levelling off at around 16% as more and more data became available. (This display is similar to Figure 4.1.) The histogram displays the proportions

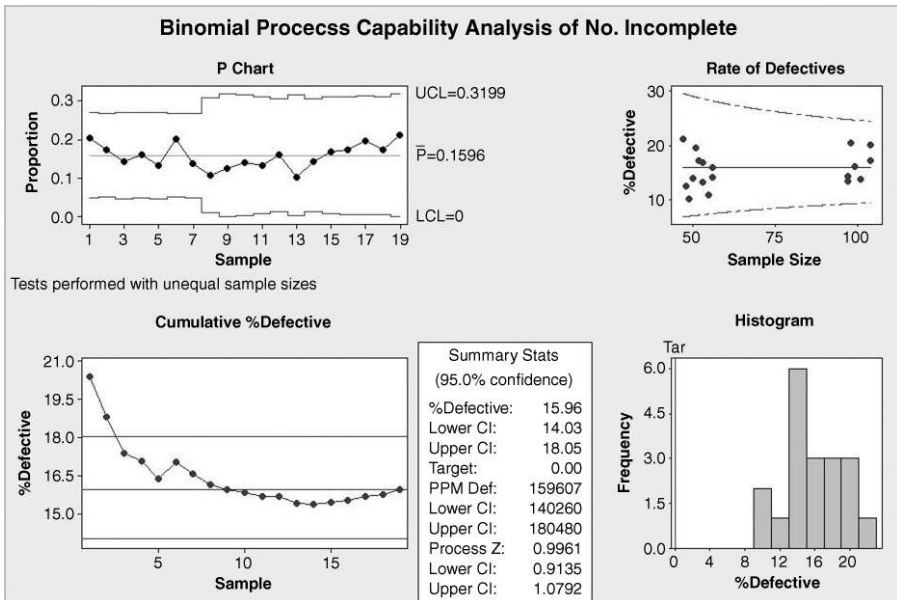


Figure 6.17 Binomial capability analysis of invoice data.

(expressed as percentages) of nonconforming invoices for the 19 samples analysed. The Rate of Defectives plot is essentially a funnel plot as described in Chapter 5. It is provided in this context in order to give insight into whether or not the proportion of nonconforming invoices is influenced by the subgroup size. Here the subgroups comprised all the invoices processed on each day. Had the group of points on the right been located at a higher level than the group of points on the left, then this could possibly have indicated a higher proportion of nonconforming invoices occurring on days when staff were working under greater pressure dealing with higher numbers of invoices. Finally, the Summary Stats table indicate that the overall proportion of nonconforming invoices is estimated at 15.96% or 159 607 ppm. This converts, via Appendix 1, to a sigma quality level of around 2.5. Alternatively, addition of 1.5 to the Process Z of 0.996 1 given in the output yields a sigma quality level of 2.5, to one decimal place. Confidence intervals are also given. (Note that Minitab refers to defectives rather than to nonconforming items.)

As an example of a situation where the Poisson model is potentially appropriate consider the data from work cell B referred to in Exercise 11 in Chapter 5. The worksheet PCB2.MTW gives counts of nonconformities on a series of samples of printed circuit boards. The subgroup size is given in the first column and the number of nonconformities found in the sample in the second. Use of **Stat > Quality Tools > Capability Analysis > Poisson...** yields the output shown in Figure 6.18. In the dialog **Defects:** was specified as Nonconformities and **Use Sizes in:** was specified as Boards. Under **Tests...** the option to **Perform all four tests** was checked.

The U chart of the data provides no evidence of any special cause behaviour affecting the process. The plot of Cumulative DPU (defects per unit) stabilizes at around 0.35, which is indicated by the middle of the three horizontal reference lines on the chart. The upper and lower reference lines correspond to upper and lower 95% confidence limits for defects per unit for the process of 0.31 and 0.40. All three values are given in the Summary Stats table. The

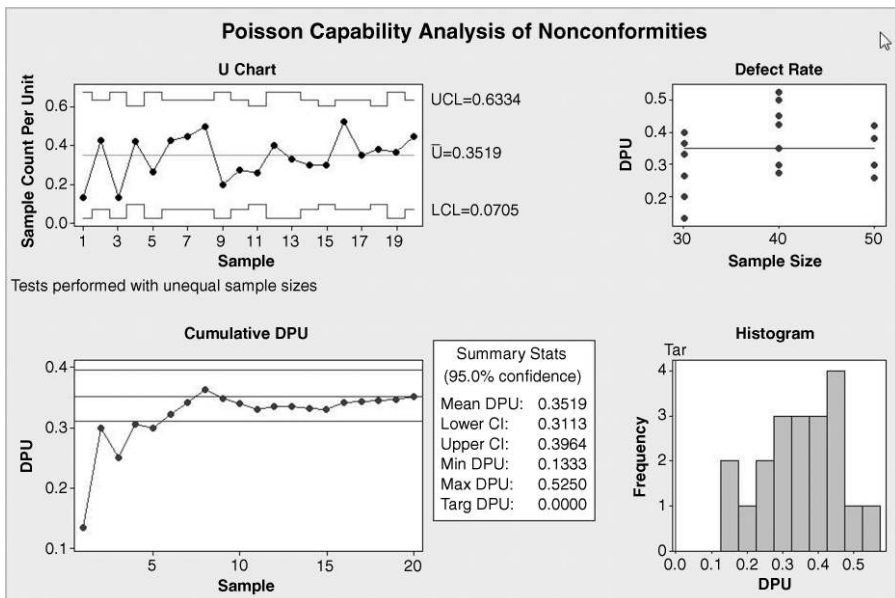


Figure 6.18 Poisson capability analysis of PCB data.

histogram shows the distribution of DPU for the subgroups, the scatterplot shows DPU plotted against subgroup size. There is a suggestion from the arch-shaped scatterplot that DPU may be influenced by subgroup size – further investigation of this would clearly be of interest to those responsible for the identification of nonconformities.

6.2 Exercises and follow-up activities

1. The specification limits for the amount of mineral water delivered by a bottling process to nominal 1.5 litre bottles are 1.480 and 1.520 litres, respectively. Forty samples of five fill volumes taken at 30-minute intervals from a bottling process are given in Volumes.MTW. Assess the capability of the process.

Stack the data into a single column and obtain a two-sided tolerance interval that covers at least 99.9% of fill volumes with 95% confidence. Move the mouse pointer to the horizontal scale in the central plot so that Y-scale is displayed in a text box. Double-click and change the scale range minimum and maximum to 1.47 and 1.53, respectively. Right-click the display and use **Add > Reference Lines...** to superimpose labelled specification limits on the plot. Note the insight into process capability that the display now provides.

2. The outside diameter (OD) of the pilot on an output shaft is an important quality characteristic. Table 6.5 gives data obtained from 25 subgroups of size 4 and records the diameter measured as deviation from nominal (micrometres). The data are also provided in the worksheet OD.MTW and are reproduced by permission of the Statistics and Actuarial Science Department, University of Waterloo, Canada from Steiner *et al.* (1997, p. 6).
 - (i) Verify, using *Xbar* and *R* control charts, that the process is behaving in a stable, predictable manner.
 - (ii) Given that the specification limits for the deviation from nominal are -20 and 20, carry out a capability analysis using both **Capability Analysis > Normal...** and **Capability Sixpack > Normal...**
 - (iii) Confirm the values $C_p = 1.29$ and $C_{pk} = 1.26$ by direct calculation using the within standard deviation estimate provided in the Process Data textbox in the output from **Capability Analysis > Normal...**
 - (iv) Confirm the values $P_p = 1.19$ and $P_{pk} = 1.16$ by direct calculation using the overall standard deviation estimate provided.

Table 6.5 Pilot diameter data.

| Sub group | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | |
|-----------|-----|-----|----|----|----|----|----|---|----|----|-----|-----|----|----|----|----|----|----|----|----|----|----|-----|----|----|---|
| x_1 | -10 | -14 | -2 | -3 | 12 | 0 | 2 | 0 | -8 | 4 | -8 | -10 | -8 | -4 | 2 | 12 | 2 | -6 | -2 | 2 | 0 | -2 | -10 | 6 | | |
| x_2 | -6 | -4 | 12 | -5 | 6 | 0 | -6 | 6 | 4 | 0 | 2 | 4 | 2 | 2 | 6 | 2 | 6 | 2 | -4 | 4 | 4 | 4 | 4 | 4 | 4 | 8 |
| x_3 | 0 | -6 | -2 | -5 | 2 | -6 | 8 | 4 | -4 | 2 | -14 | -10 | -4 | 0 | 0 | 0 | 0 | 2 | 0 | 2 | 2 | -2 | -12 | -4 | | |
| x_4 | 0 | 4 | 8 | -1 | 2 | -8 | -6 | 8 | 8 | 2 | 6 | 6 | 4 | 4 | 8 | 2 | 2 | -8 | 0 | 4 | 6 | 4 | 4 | 4 | 4 | 2 |

- (v) Using the estimate of process 'fall-out' in ppm based on the within standard deviation estimate, obtain an estimate of the sigma quality level of the process using Appendix 1. Use the **Benchmark Z's (sigma level)** option with **Capability Analysis > Normal...** to confirm your estimate.
- (vi) Change the data set by *subtracting* 1 from each observation in subgroup 4 and *adding* 10 to each observation in subgroup 15. Repeat the capability analysis and observe that the within estimate of standard deviation is unchanged since the sample ranges are unchanged, but that the overall estimate has increased. As a consequence, C_p is unchanged but P_p is reduced. Some authors argue that this means that P_p and P_{pk} measure how the process *actually* performed, while C_p and C_{pk} measure how the process *could* perform. However, scrutiny of Xbar and R charts of the modified data reveals that they correspond to a process that is not stable and predictable.
3. Door to needle time (DTN) is the time from arrival at hospital when the ambulance stops outside the hospital (door) to the start of the thrombolytic treatment (needle) for patients with an acute myocardial infarction. The health authority responsible for management of the hospital has an upper specification limit of 30 minutes for DTN for such patients. Given the sample of door to needle times in the worksheet DTN1.MTW, carry out a capability analysis. Following process changes, a further sample of door to needle times was recorded and is available in the worksheet DTN2.MTW. Assess the impact of the changes on process capability. Obtain appropriate before and after tolerance intervals that cover 99% of DTN times with 95% confidence and note the insight that these provide into the impact of the process changes.
4. The worksheet Sand.MTW contains the percentage of sand by weight in samples of aggregate material for use in the construction industry. Given that the upper specification limit is 10%, carry out a capability analysis of the data. You should verify that direct use of Capability Analysis (Normal) overestimates capability compared with that obtained using a Box-Cox transformation and with that obtained using a Weibull distribution.
5. A large call centre, which operates from 08.00 until 20.00 Monday to Friday, is staffed by teams A, B and C, with A responsible for 08.00 until 12.00, B for 12.00 until 16.00 and C for the remaining period each day. For one particular week recordings of samples of 40 of the calls received during each half-hour period were analysed by supervisors for conformance to specifications. The worksheet Calls.MTW gives summary data.
- (i) Carry out a binomial capability analysis of the complete data set without taking team into account. Note the signals on the P chart and the oscillatory behaviour of the Cumulative %Defective (nonconforming) plot.
- (ii) Unstack the data by team and carry out a binomial capability analysis for each team. What do you conclude?
6. Set up the funnel plot data in Table 5.7 in Minitab and, viewing the data as a series of samples from a process, carry out a binomial capability analysis. Compare the funnel plot in the output with that in Figure 5.29.